The Fundamental Theorem of Algebra

Theorem:
If $f(x)$ is a polynomial function of degree n with leading term $a_n x^n$, then f has precisely n linear factors of the form.
$f(x) = a_n(x - c_1)(x - c_2)...(x - c_n)$
where $c_1, c_2, ..., c_n$ are complex numbers (not necessarily unique).

What does this theorem say about the polynomial
$f(x) = x^3 + 2x^2 - 2x - 4$?
Hint: $-2,\ \sqrt{2},\ -\sqrt{2}$ are the only zeros of f.

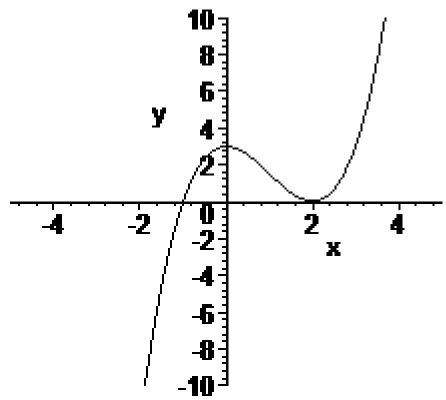
Definition:
If $(x-c)^n$ is a factor of a polynomial, then c is said to be a zero of the polynomial with **multiplicity n.** *To determine the multiplicity of a zero of a function you need to know all of the factors of the function. The graph of a function can also give you insight into the zeros of a function and their multiplicities.*

$f(x) = \frac{3}{4}(x+1)(x-2)^2$;

— 1 and 1 are zeros of f.

— 1 is a zero of multiplicity 1.

2 is a zero of f with multiplicity 2.

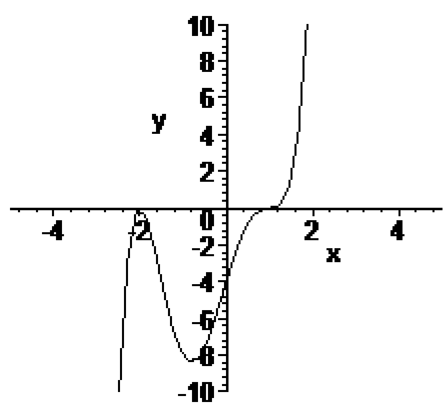


$f(x) = (x+2)^2(x-1)^3$;

— 2 and 1 are zeros of f.

— 2 is a zero of multiplicity 2.

1 is a zero of f with multiplicity 3.



Geometric Meaning of Multiplicity:

If c is a zero of multiplicity n then:

1) When n is odd the graph of the polynomial will cross the x-axis at $(c, 0)$.

2) When n is even the graph of the polynomial will touch the x-axis at $(c, 0)$ but will not pass through.

3) When $n \geq 2$ then the graph of f will "flatten out" as it approaches $(c, 0)$

If $g(x) = 2(x-1)(x+1)^2(x+2)^3$

Determine the degree of g.

Determine the y-intercept.

Determine the x-intercepts.

Where does the graph of g cross the x-axis?

Where does the graph of g "bounce off" of the x-axis?

For which $x$intercepts does the graph of g "flatten out" ?

Use this information and the end-behaviors to draw a rough sketch of the graph of g .

Completely factor the polynomial function
$P(x) = x^4 - x^3 + 2x^2 - 4x - 8$ and find all of the zeros of P.

Theorem:
If the imaginary number $a+bi$ is a zero of a polynomial then its conjugate $a-bi$ is also a zero.

If $5-3i$ is a zero of the polynomial function $f(x)=x^4-6x^3-11x^2+186x-170$, find all of the zeros of f and write f in factored form.

Construct a 5th degree polynomial with a leading coefficient of 4 that has 2 as a zero with multiplicity of 2 and -3 is the only other zero.